Chapter 11 Review – Math 101

#1. $x^2 - 4x + 1 = 0$

$-1 + x^2 - 4x + 1 = -1 + 0$

$x^2 - 4x = -1$

$x^2 - 4x + \underline{4} = -1 + \underline{4}$

$(x - 2)(x - 2) = 3$

$(x-2)^2 = 3$

Either

$x - 2 = +\sqrt{3}$, or $x - 2 = -\sqrt{3}$

$2 + x - 2 = 2 + \sqrt{3}$ | $2 + x - 2 = 2 + (-\sqrt{3})$

$x = 2 + \sqrt{3}$ | $x = 2 - \sqrt{3}$

$\{2+\sqrt{3}, 2-\sqrt{3}\}$

SDWK

$\left[\frac{1}{2}(-4)\right]^2 = (-2)^2$

$= 4$

#2 $2x^2 + 3x - 4 = 0$

$\frac{1}{2}[2x^2 + 3x - 4] = \frac{1}{2} \cdot 0$

$x^2 + \frac{3}{2}x - 2 = 0$

$2 + x^2 + \frac{3}{2}x - 2 = 2 + 0$

$x^2 + \frac{3}{2}x = 2$

$x^2 + \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16}$

$(x + \frac{3}{4})(x + \frac{3}{4}) = \frac{32}{16} + \frac{9}{16}$

$(x + \frac{3}{4})^2 = \frac{41}{16}$

Either

$x + \frac{3}{4} = +\sqrt{\frac{41}{16}}$, or $x + \frac{3}{4} = -\sqrt{\frac{41}{16}}$

$x + \frac{3}{4} = \frac{\sqrt{41}}{\sqrt{16}}$ | $x + \frac{3}{4} = -\frac{\sqrt{41}}{\sqrt{16}}$

$x + \frac{3}{4} = \frac{\sqrt{41}}{4}$ | $x + \frac{3}{4} = -\frac{\sqrt{41}}{4}$

SDWK

$\left[\frac{1}{2}\left(\frac{3}{2}\right)\right]^2 = \left(\frac{3}{4}\right)^2$

$= \frac{9}{16}$

$-\frac{3}{4} + x + \frac{3}{4} = -\frac{3}{4} + \frac{\sqrt{41}}{4}$

$x = \frac{-3 + \sqrt{41}}{4}$

or

$-\frac{3}{4} + x + \frac{3}{4} = -\frac{3}{4} - \frac{\sqrt{41}}{4}$

$x = \frac{-3 - \sqrt{41}}{4}$

$\left\{\frac{-3-\sqrt{41}}{4}, \frac{-3+\sqrt{41}}{4}\right\}$

Chapter 11 Review — Math 101 —

#3. $3x^2 - 10 = 65$

$10 + 3x^2 - 10 = 10 + 65$

$3x^2 = 75$

$\frac{3x^2}{3} = \frac{75}{3}$

$x^2 = 25$

Either

$x = +\sqrt{25}$, or $x = -\sqrt{25}$

$x = 5$ $\quad x = -5$

$\{-5, 5\}$

#4. $2x^2 + 10 = -8$

$-10 + 2x^2 + 10 = -8 + (-10)$

$2x^2 = -18$

$\frac{2x^2}{2} = \frac{-18}{2}$

$x^2 = -9$

Either

$x = +\sqrt{-9}$, or $x = -\sqrt{-9}$

$x = \sqrt{9}\sqrt{-1}$ $\quad x = -\sqrt{9}\sqrt{-1}$

$x = 3i$ $\quad x = -3i$

$\{-3i, 3i\}$

$i = \sqrt{-1}$

Chapter 11 Reviews — Math 101 —

#5 $(2x-3)^2 - 7 = 0$

$7 + (2x-3)^2 - 7 = 7 + 0$

$(2x-3)^2 = 7$

either

$2x-3 = +\sqrt{7}$

$3 + 2x - 3 = 3 + \sqrt{7}$

$\frac{2x}{2} = \frac{3+\sqrt{7}}{2}$

$x = \frac{3+\sqrt{7}}{2}$

or $2x-3 = -\sqrt{7}$

$3 + 2x - 3 = 3 + (-\sqrt{7})$

$2x = 3 - \sqrt{7}$

$\frac{1}{2}(2x) = \frac{1}{2}(3-\sqrt{7})$

$x = \frac{3-\sqrt{7}}{2}$

$\left\{ \frac{3+\sqrt{7}}{2}, \frac{3-\sqrt{7}}{2} \right\}$

#6. $x^2 + 2x = 8$

$\left[\frac{1}{2}(2)\right]^2 = (1)^2 = 1$

$x^2 + 2x + 1 = 8 + 1$

$(x+1)(x+1) = 9$

$(x+1)^2 = 9$

either

$x+1 = +\sqrt{9}$, or $x+1 = -\sqrt{9}$

$x+1 = 3$

$-1 + x + 1 = -1 + 3$

$x = 2$

$x+1 = -3$

$-1 + x + 1 = -1 + (-3)$

$x = -4$

$\{-4, 2\}$

Chapter 11 Review — Math 101 —

#7 $2x^2 = 5x + 3$

$-5x - 3 \quad -5x - 3$

$2x^2 - 5x - 3 = 0$

$\begin{cases} a = 2 \\ b = -5 \\ c = -3 \end{cases}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$x = \dfrac{5 \pm \sqrt{25 + 24}}{4}$

$x = \dfrac{5 \pm \sqrt{49}}{4}$

$x = \dfrac{5 \pm 7}{4}$

Either

$x = \dfrac{5+7}{4}$, or $x = \dfrac{5-7}{4}$

$x = \dfrac{12}{4}$ | $x = \dfrac{-2}{4}$

$x = 3$ | $x = -\dfrac{1}{2}$

$\left\{-\frac{1}{2}, 3\right\}$

By factoring

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

Either

$2x + 1 = 0$, or $x - 3 = 0$

$-1 + 2x + 1 = -1 + 0$ | $3 + x - 3 = 0 + 3$

$2x = -1$ | $x = 3$

$\dfrac{2x}{2} = -\dfrac{1}{2}$

$x = -\dfrac{1}{2}$

$\left\{-\frac{1}{2}, 3\right\}$

Chapter 11 Review — Math 101 —

#8 $2x^2+7x=-1$

$1+2x^2+7x=-1+1$

$2x^2+7x+1=0$

$\begin{cases} a=2 \\ b=7 \\ c=1 \end{cases}$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(7)\pm\sqrt{(7)^2-4(2)(1)}}{2(2)}$

$x=\dfrac{-7\pm\sqrt{49-8}}{4}$

$x=\dfrac{-7\pm\sqrt{41}}{4}$

$\left\{\dfrac{-7+\sqrt{41}}{4}, \dfrac{-7-\sqrt{41}}{4}\right\}$

#9 $1+\dfrac{3}{x}=\dfrac{-1}{x^2}$

$\dfrac{x^2}{1}\left[1+\dfrac{3}{x}\right]=\dfrac{x^2}{1}\left(\dfrac{-1}{x^2}\right)$

$\dfrac{x^2}{1}\cdot 1+\dfrac{x^2}{1}\cdot\dfrac{3}{x}=-1$

$x^2+3x=-1$

$1+x^2+3x=-1+1$

$x^2+3x+1=0$

$\begin{cases} a=1 \\ b=3 \\ c=1 \end{cases}$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(3)\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$

$x=\dfrac{-3\pm\sqrt{9-4}}{2}$

$x=\dfrac{-3\pm\sqrt{5}}{2}$

$\left\{\dfrac{-3+\sqrt{5}}{2}, \dfrac{-3-\sqrt{5}}{2}\right\}$

Chapter 11 Review – Math 101

#10. $3x^2 - 2x + 4 = 0$

$a = 3$, $b = -2$, $c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 - 48}}{6}$$

$$x = \frac{2 \pm \sqrt{-44}}{6}$$

$$x = \frac{2 \pm \sqrt{4}\sqrt{11}\sqrt{-1}}{6}$$

$$x = \frac{2 \pm 2i\sqrt{11}}{6}$$

$$x = \frac{2}{6} \pm \frac{2i\sqrt{11}}{6}$$

$$x = \frac{1}{3} \pm \frac{i\sqrt{11}}{3}$$

$$\left\{ \frac{1}{3} + \frac{i\sqrt{11}}{3}, \frac{1}{3} - \frac{i\sqrt{11}}{3} \right\}$$

#11. $x^2 + x + 1 = 0$

$a = 1$, $b = 1$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-1 \pm \sqrt{3}\sqrt{-1}}{2}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$

$$\left\{ -\frac{1}{2} + \frac{i\sqrt{3}}{2}, -\frac{1}{2} - \frac{i\sqrt{3}}{2} \right\}$$

7/31

Chapter 11 Review — Math 107 —

#12 $x^4 + 5x^2 = 36$

Let $t = x^2$, $t^2 = (x^2)^2 = x^4$

by substitution, we get the following:

$t^2 + 5t = 36$

$-36 + t^2 + 5t = -36 + 36$

$t^2 + 5t - 36 = 0$

$(t - 4)(t + 9) = 0$

either

$t - 4 = 0$, or $t + 9 = 0$

$4 + t - 4 = 4 + 0$

$\boxed{t = 4}$

$x^2 = 4$

either

$x = +\sqrt{4}$, or $x = -\sqrt{4}$

$x = 2$ | $x = -2$

$-9 + t + 9 = -9 + 0$

$\boxed{t = -9}$

$x^2 = -9$

either

$x = +\sqrt{-9}$, or $x = \sqrt{-9}$

$x = \sqrt{9}\sqrt{-1}$ | $x = -\sqrt{9}\sqrt{-1}$

$x = 3i$ | $x = -3i$

SDWK

36
1, 36
2, 18
3, 12
4, 9
6, 6

$\{2, -2, 3i, -3i\}$

8/31

chapter 11 Review — Math 101 —

#13. $x - 6\sqrt{x} = 7$

let $t = \sqrt{x}$, $t^2 = (\sqrt{x})^2 = x$

by substitution, we get the following:

$t^2 - 6t = 7$

$-7 + t^2 - 6t = -7 + 7$

$t^2 - 6t - 7 = 0$

$(t - 7)(t + 1) = 0$

Either

$t - 7 = 0$, or $t + 1 = 0$

$7 + t - 7 = 7 + 0$ | $-1 + t + 1 = -1 + 0$

$t = 7$ | $t = -1$

$\sqrt{x} = 7$ | $\sqrt{x} = -1$

$(\sqrt{x})^2 = (7)^2$ | $(\sqrt{x})^2 = (-1)^2$

$x = 49$ | $x = 1$

check:

$x = 49$

$(49) - 6\sqrt{(49)} = 7$

$49 - 6 \cdot 7 = 7$

$49 - 42 = 7$

$7 = 7$ TRUE!

$x = 1$

$(1) - 6\sqrt{(1)} = 7$

$1 - 6 = 7$

$-5 = 7$

false!

{49}

9/31

Chapter 11 Reviews — Math 101 —

#14

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 12 = 0$$

let $t = x^{1/3}$, $t^2 = (x^{1/3})^2 = x^{2/3}$

by substitution, we get the following:

$$t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0$$

either

$t - 4 = 0$, or $t + 3 = 0$

$4 + t - 4 = 4 + 0$ | $-3 + t + 3 = -3 + 0$

$t = 4$ | $t = -3$

$x^{1/3} = 4$ | $x^{1/3} = -3$

$(x^{1/3})^3 = (4)^3$ | $(x^{1/3})^3 = (-3)^3$

$x = 64$ | $x = -27$

$\{64, -27\}$

Divide

12

1, 12

2, 6

3, 4

check

$x = 64$

$(64)^{2/3} - (64)^{1/3} - 12 = 0$

$(\sqrt[3]{64})^2 - \sqrt[3]{64} - 12 = 0$

$(4)^2 - 4 - 12 = 0$

$16 - 4 - 12 = 0$

$12 - 12 = 0$

$0 = 0$

TRUE!

$x = -27$

$(-27)^{2/3} - (-27)^{1/3} - 12 = 0$

$(\sqrt[3]{-27})^2 - \sqrt[3]{-27} - 12 = 0$

$(-3)^2 - (-3) - 12 = 0$

$9 + 3 - 12 = 0$

$12 - 12 = 0$

$0 = 0$

TRUE!

10/31

Chapter 11 Reviews — Math 101 —

#15

$$(x-1)^2+(x-1)+1=0$$

let $t=x-1$, $t^2=(x-1)^2$

by substitution, we get the following:

$$t^2+t+1=0$$

$a=1$, $b=1$, $c=1$

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$t=\frac{-(1)\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$

$$t=\frac{-1\pm\sqrt{1-4}}{2}$$

$$t=\frac{-1\pm\sqrt{-3}}{2}$$

either

$$t=\frac{-1+\sqrt{3}\sqrt{-1}}{2}, \text{ or } t=\frac{-1-\sqrt{3}\sqrt{-1}}{2}$$

$$\boxed{t=\frac{-1+i\sqrt{3}}{2}}, \text{ or } \boxed{t=\frac{-1-i\sqrt{3}}{2}}$$

$$x-1=\frac{-1+i\sqrt{3}}{2}$$

$$1+x-1=\frac{2}{2}+\left(\frac{-1}{2}\right)+\frac{i\sqrt{3}}{2}$$

$$x=\frac{1}{2}+\frac{i\sqrt{3}}{2}$$

$$x-1=\frac{-1-i\sqrt{3}}{2}$$

$$1+x-1=\frac{2}{2}+\left(\frac{-1}{2}\right)-\frac{i\sqrt{3}}{2}$$

$$x=\frac{1}{2}-\frac{i\sqrt{3}}{2}$$

$$\left\{\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}-\frac{i\sqrt{3}}{2}\right\}$$

11/31

Chapter 11 Review – Math 101

#16 $f(x) = x^2 - 4$, $a=1$, $b=0$, & $c=-4$

Find x-intercepts: $y=0$

$0 = x^2 - 4$

$0 = (x-2)(x+2)$

either | or

$x-2=0$ | $x+2=0$

$x=2$ | $x=-2$

$(2,0)$ & $(-2,0)$

Find y-intercept: $x=0$

$f(0) = (0)^2 - 4$

$f(0) = -4$

$(0,-4)$

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(0)}{2(1)}$

$x = 0$

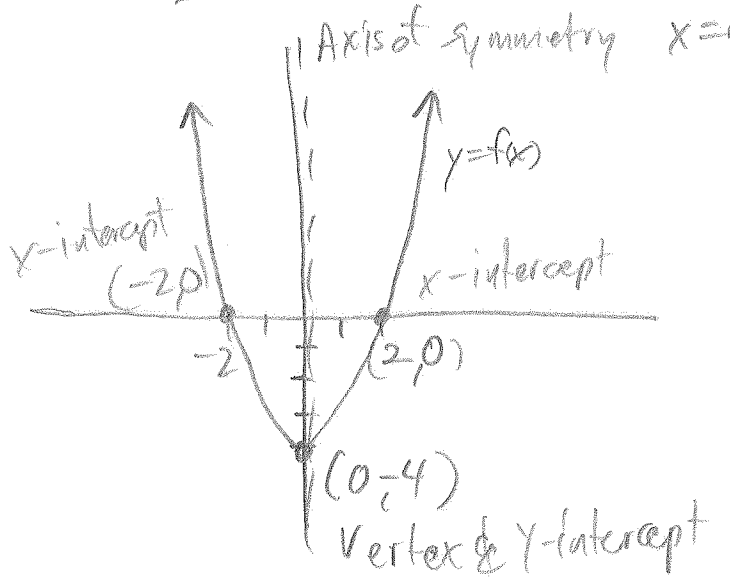
$y = f\left(\frac{-b}{2a}\right)$

$y = f(0)$

$y = (0)^2 - 4$

$y = -4$

$(0,-4)$



or

$f(x) = x^2 - 4$

$f(x) = 1 \cdot (x-0)^2 - 4$

$f(x) = a(x-h)^2 + k$

$a=1$, $a>0$, up

$h=0$, $k=-4$

(h,k) = Vertex

$(0,-4)$ = Vertex

Chapter 11 Review – Material

#17 $f(x) = (x-1)^2 + 2$

$f(x) = a(x-h)^2 + k$

$a = 1$, $a > 0$ up

$h = 1$, $k = 2$

(h, k) = Vertex

$(1, 2)$ = Vertex ✓

Find x-intercepts: $y = 0$

$0 = (x-1)^2 + 2$

$-2 + 0 = -2 + (x-1)^2 + 2$

$-2 = (x-1)^2$

Either

$\sqrt{-2} = x - 1$

$\sqrt{2}\,i = x - 1$

$1 + \sqrt{2}\,i = 1 + x - 1$

$1 + i\sqrt{2} = x$

↑ complex

, or $-\sqrt{-2} = x - 1$

$-\sqrt{2}\,i = x - 1$

$1 + (-i\sqrt{2}) = 1 + x - 1$

$1 - i\sqrt{2} = x$

↑ , No Real solutions

No x-intercepts

Find y-intercepts: $x = 0$

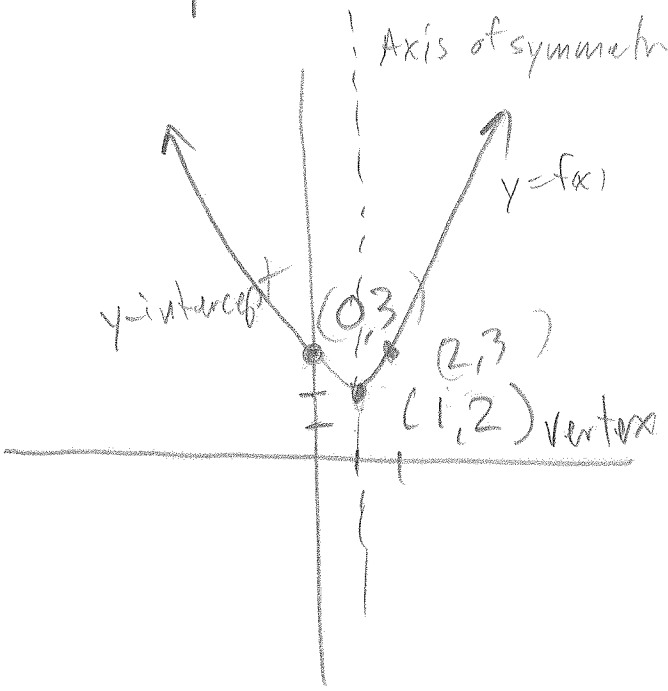
$f(0) = [(0) - 1]^2 + 2$

$f(0) = (-1)^2 + 2$

$f(0) = 1 + 2$

$f(0) = 3$

$(0, 3)$



Chapter 11 Review — Math 101

#18. $f(x) = -(x+2)^2 - 4$ $\quad a = -1$, $a < 0$, down

$f(x) = a(x-h)^2 + k$ $\quad h = -2$, $k = -4$

(h,k) = Vertex

$(-2,-4)$ = Vertex

Find x-intercepts; $y = 0$

$0 = -(x+2)^2 - 4$

$(x+2)^2 + 0 = -(x+2)^2 - 4 + (x+2)^2$

$(x+2)^2 = -4$

Either

$x+2 = +\sqrt{-4}$, or $x+2 = -\sqrt{-4}$

$x+2 = \sqrt{4}\sqrt{-1}$ | $x+2 = -\sqrt{4}\sqrt{-1}$

$x+2 = 2i$ | $x+2 = -2i$

$-2+x+2 = -2+2i$ | $-2+x+2 = -2+(-2i)$

$x = -2+2i$ | $x = -2-2i$

↑ complex, NO Real Solutions

No x-intercepts!

Find y-intercepts; $x = 0$

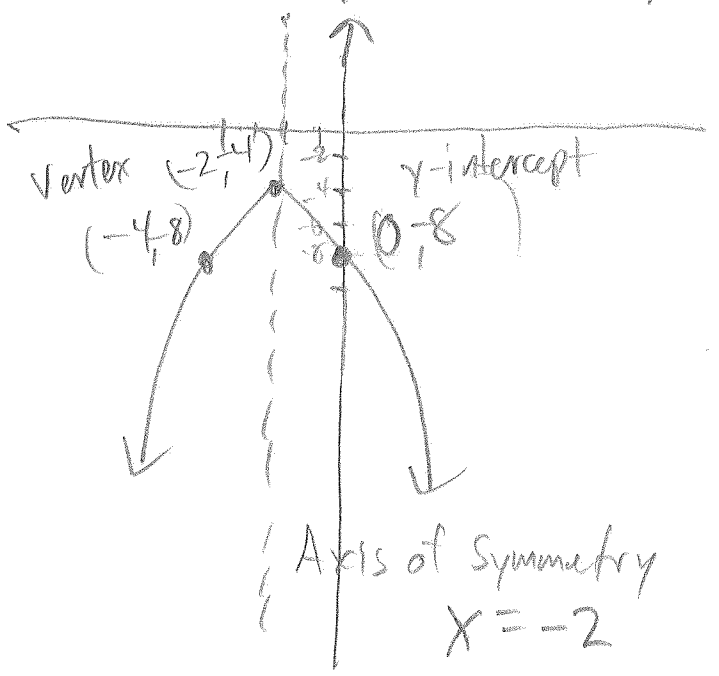
$f(0) = -[(0)+2]^2 - 4$

$f(0) = -(2)^2 - 4$

$f(0) = -4 - 4$

$f(0) = -8$

$(0,-8)$



Chapter 11 Review — Math 101

#19 $f(x) = x^2 + 2x - 8$ $\quad a > 0$, up

$a = 1$, $b = 2$ & $c = -8$

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(2)}{2(1)}$

$x = -1$

$y = f\left(\frac{-b}{2a}\right)$

$y = f(-1)$

$y = (-1)^2 + 2(-1) - 8$

$y = 1 - 2 - 8$

$y = -1 - 8$

$y = -9$

$(-1, -9)$ = Vertex

Find x-intercepts: $y = 0$

$0 = x^2 + 2x - 8$

$0 = (x - 2)(x + 4)$

Either

$x - 2 = 0$, or $x + 4 = 0$

$2 + x - 2 = 2 + 0$ | $-4 + x + 4 = -4 + 0$

$x = 2$ | $x = -4$

$(2, 0)$ & $(-4, 0)$

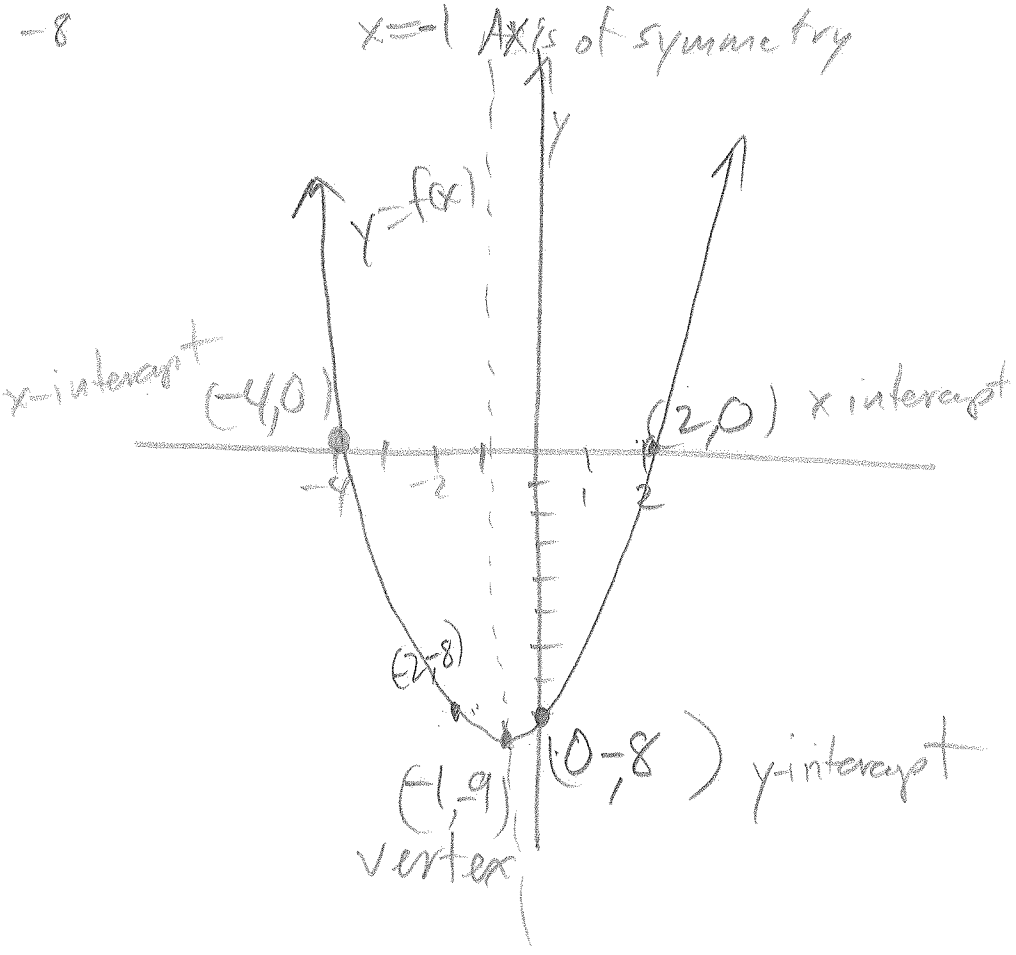
Find y-intercept: $x = 0$

$f(0) = (0)^2 + 2(0) - 8$

$f(0) = 0 - 8$

$f(0) = -8$

$(0, -8)$



Chapter 11 Review — Math 101

#20 $f(x) = 2x^2 - 5x - 3$ $\quad a > 0$, up

$a = 2$, $b = -5$ & $c = -3$

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(-5)}{2(2)}$

$x = \frac{5}{4}$

$f\left(\frac{-b}{2a}\right) = y$

$f\left(\frac{5}{4}\right) = y$

$2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) - 3 = y$

$2\left(\frac{25}{16}\right) - \frac{25}{4} - 3 = y$

$\frac{25}{8} - \frac{25}{4}\cdot\frac{2}{2} - \frac{3}{1}\cdot\frac{8}{8} = y$

$\frac{25}{8} - \frac{50}{8} - \frac{24}{8} = y$

$-\frac{25}{8} - \frac{24}{8} = y$

$-\frac{49}{8} = y$

Vertex $= \left(\frac{5}{4}, \frac{-49}{8}\right)$

Find x-intercepts: $y = 0$

$0 = 2x^2 - 5x - 3$

$0 = (2x + 1)(x - 3)$

Either

$2x + 1 = 0$, or $x - 3 = 0$

$-1 + 2x + 1 = -1 + 0$ | $3 + x - 3 = 3 + 0$

$2x = -1$ | $x = 3$

$\frac{2x}{2} = \frac{-1}{2}$

$x = \frac{-1}{2}$

$\left(-\frac{1}{2}, 0\right)$ $\quad (3, 0)$

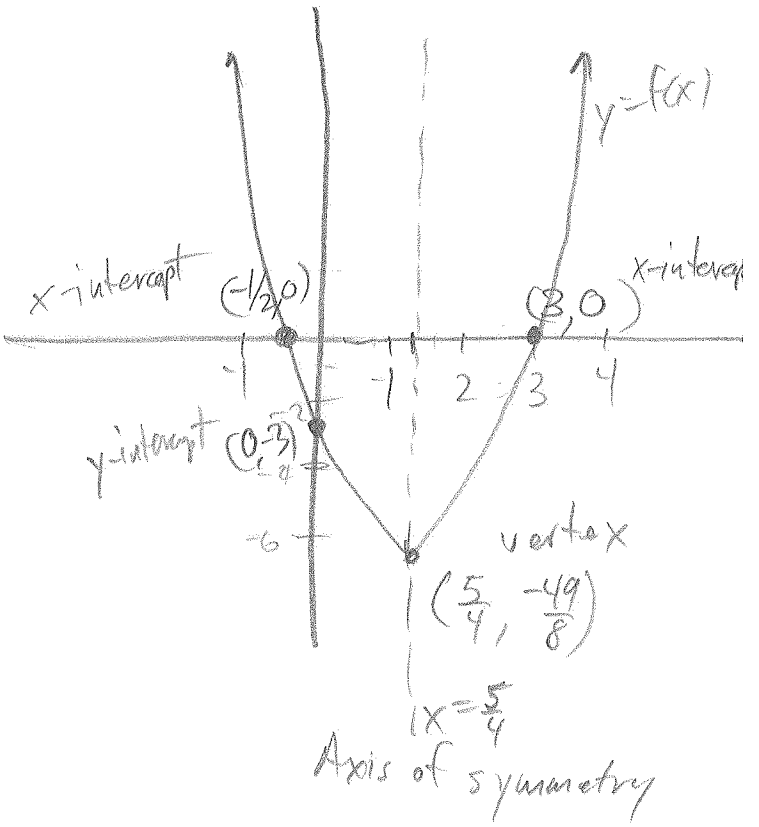
Find y-intercept: $x = 0$

$f(0) = 2(0)^2 - 5(0) - 3$

$f(0) = 0 - 3$

$f(0) = -3$

$(0, -3)$



Chapter 11 Review — Math 101 —

#21 $f(x) = x^2 + 2x + 2$ $a > 0$, up

$a = 1$, $b = 2$, & $c = 2$

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(2)}{2(1)}$

$x = -1$

$y = f\left(\frac{-b}{2a}\right)$

$y = f(-1)$

$y = (-1)^2 + 2(-1) + 2$

$y = 1 - 2 + 2$

$y = 1$

$(-1, 1)$ = Vertex

Find x-intercepts: $y = 0$

$0 = x^2 + 2x + 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 - 8}}{2}$

$x = \frac{-2 \pm \sqrt{-4}}{2}$

$x = \frac{-2 \pm \sqrt{4}\sqrt{-1}}{2}$

$x = \frac{-2 \pm 2i}{2}$

$x = \frac{-2}{2} \pm \frac{2i}{2}$

$x = -1 \pm i$

complex, No Real Solutions, No X-intercepts!

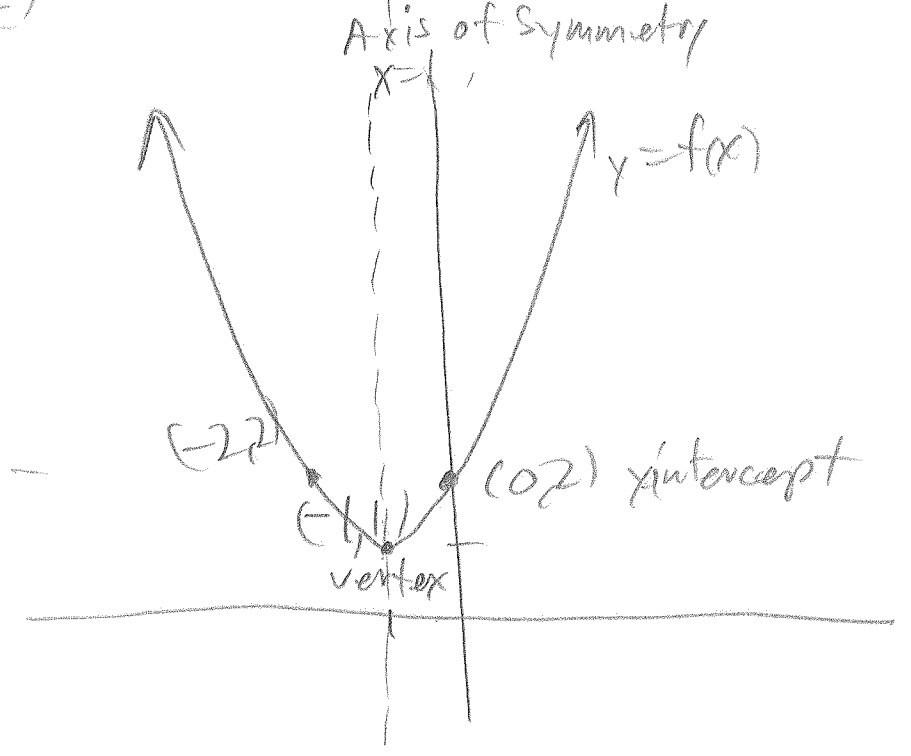
Find y-intercepts: $x = 0$

$f(0) = (0)^2 + 2(0) + 2$

$f(0) = 0 + 2$

$f(0) = 2$

$(0, 2)$



Chapter 11 Review — Math 101 —

#22: $2x^2 - 5x - 3 \leq 0$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

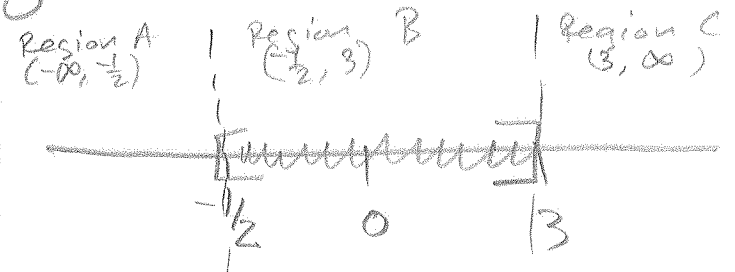
Either

$2x + 1 = 0$, or $x - 3 = 0$

$-1 + 2x + 1 = -1 + 0$ | $3 + x - 3 = 3 + 0$

$\frac{2x}{2} = \frac{-1}{2}$ | $x = 3$

$x = -\frac{1}{2}$



Test $x = -1$	Test $x = 0$	Test $x = 4$
$2(-1)^2 - 5(-1) - 3 \leq 0$	$2(0)^2 - 5(0) - 3 \leq 0$	$2(4)^2 - 5(4) - 3 \leq 0$
$2(1) + 5 - 3 \leq 0$	$0 - 0 - 3 \leq 0$	$2 \cdot 16 - 20 - 3 \leq 0$
$2 + 5 - 3 \leq 0$	$-3 \leq 0$	$32 - 20 - 3 \leq 0$
$7 - 3 \leq 0$	TRUE	$12 - 3 \leq 0$
$4 \leq 0$		$9 \leq 0$
False		False

Solution Set in Interval form = $\left[-\frac{1}{2}, 3\right]$

#23: $x^2 - 9 \geq 0$

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

Either

$x - 3 = 0$, or $x + 3 = 0$

$3 + x - 3 = 3 + 0$ | $-3 + x + 3 = -3 + 0$

$x = 3$ | $x = -3$

Test $x = -4$	Test $x = 0$	Test $x = 4$
$(-4)^2 - 9 \geq 0$	$(0)^2 - 9 \geq 0$	$(4)^2 - 9 \geq 0$
$16 - 9 \geq 0$	$-9 \geq 0$	$16 - 9 \geq 0$
$7 \geq 0$	False.	$7 \geq 0$
TRUE!		TRUE!

Solution Set in Interval form = $(-\infty, -3] \cup [3, \infty)$

Chapter 11 Review – Math 101 –

#24: $9x^2+3x-2 \geq 0$

$9x^2+3x-2=0$

$(3x-1)(3x+2)=0$

Either

$3x-1=0$	or $3x+2=0$
$1+3x-1=1+0$	$-2+3x+2=-2+0$
$3x=1$	$3x=-2$
$\frac{3x}{3}=\frac{1}{3}$	$\frac{3x}{3}=\frac{-2}{3}$
$x=\frac{1}{3}$	$x=\frac{-2}{3}$

Region A $(-\infty, -\frac{2}{3})$ | Region B $(-\frac{2}{3}, \frac{1}{3})$ | Region C $(\frac{1}{3}, \infty)$

Test $x=-1$	Test $x=0$	Test $x=1$
$9(-1)^2+3(-1)-2 \geq 0$	$9(0)^2+3(0)-2 \geq 0$	$9(1)^2+3(1)-2 \geq 0$
$9\cdot1-3-2 \geq 0$	$0-2 \geq 0$	$9\cdot1+3-2 \geq 0$
$9-3-2 \geq 0$	$-2 \geq 0$	$9+3-2 \geq 0$
$6-2 \geq 0$	False!	$12-2 \geq 0$
$4 \geq 0$		$10 \geq 0$
TRUE!		TRUE!

Solution Set in Interval Form $= (-\infty, -\frac{2}{3}) \cup (\frac{1}{3}, \infty)$

#25 $\frac{x+5}{x+2} < 0$

$\frac{x+5}{x+2}=0$

$x+5=0$	$x+2=0$
$-5+x+5=-5+0$	$-2+x+2=-2+0$
$x=-5$	$x=-2$

Region A $(-\infty, -5)$ | Region B $(-5, -2)$ | Region C $(-2, \infty)$

Test $x=-6$	Test $x=-3$	Test $x=0$
$\frac{(-6)+5}{(-6)+2} < 0$	$\frac{(-3)+5}{(-3)+2} < 0$	$\frac{(0)+5}{(0)+2} < 0$
$\frac{-1}{-4} < 0$	$\frac{2}{-1} < 0$	$\frac{5}{2} < 0$
$\frac{1}{4} < 0$	$-2 < 0$	False!
False!	TRUE!	

Solution Set in Interval Form $= (-5, -2)$

Chapter 11 Review – Math 101 –

#26:

$$\frac{x+1}{x+3} < 2$$

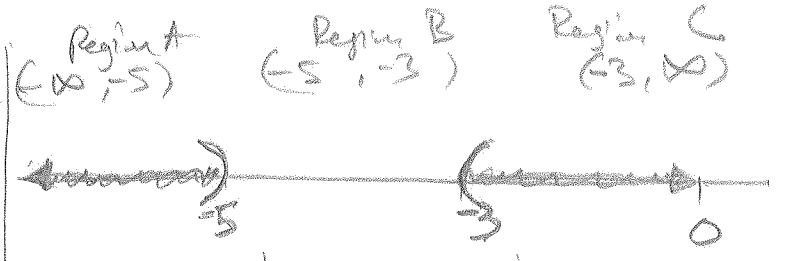
$$-2 + \frac{x+1}{x+3} < -2+2$$

$$\frac{-2}{1} \cdot \frac{(x+3)}{(x+3)} + \frac{x+1}{x+3} < 0$$

$$\frac{-2x-6+x+1}{x+3} < 0$$

$$\boxed{\frac{-x-5}{x+3} < 0}$$

$-x-5=0$	$x+3=0$
$-x-5+5=5+0$	$-3+x+3=-3+0$
$-x=5$	$x=-3$
$-1\cdot(-x)=-1\cdot 5$	
$x=-5$	



Test $x=-6$	Test $x=-4$	Test $x=0$
$\frac{(-6)+1}{(-6)+3} < 2$	$\frac{(-4)+1}{(-4)+3} < 2$	$\frac{(0)+1}{(0)+3} < 2$
$\frac{-5}{-3} < 2$	$\frac{-3}{-1} \not< 2$	$\frac{1}{3} < 2$
$\frac{5}{3} < 2$	$3 < 2$	TRUE!
TRUE!	False!	

Solution Set in Interval Form

$= (-\infty,-5) \cup (-3,\infty)$

Chapter 11 Review — Math 10C —

#27.

a. 4, −5

$x = 4$ or $x = -5$

$-4 + x = -4 + 4$ | $5 + x = -5 + 5$

$x - 4 = 0$ | $x + 5 = 0$

$(x-4)(x+5) = 0$

$x^2 + 5x - 4x - 20 = 0$

$x^2 + x - 20 = 0$ ✓

b. $\sqrt{3}$, $-\sqrt{3}$

$x = \sqrt{3}$ or $x = -\sqrt{3}$

$-\sqrt{3} + x = -\sqrt{3} + \sqrt{3}$ | $x + \sqrt{3} = -\sqrt{3} + \sqrt{3}$

$x - \sqrt{3} = 0$ | $x + \sqrt{3} = 0$

$(x-\sqrt{3})(x+\sqrt{3}) = 0$

$x^2 + \sqrt{3}x - \sqrt{3}x - \sqrt{9} = 0$

$x^2 - 3 = 0$ ✓

c. $3i$, $-3i$

$x = 3i$ or $x = -3i$

$-3i + x = -3i + 3i$ | $x + 3i = 3i + 3i$

$x - 3i = 0$ | $x + 3i = 0$

$(x-3i)(x+3i) = 0$

$x^2 + 3ix - 3ix - 9i^2 = 0$

$x^2 - 9(-1) = 0$

$x^2 + 9 = 0$ ✓

Chapter 11 Review – Math 101 –

#27

d. $\frac{1}{2}, -\frac{2}{5}$

$x = \frac{1}{2}$, or $x = -\frac{2}{5}$

$2 \cdot x = 2 \cdot \frac{1}{2}$	$5 \cdot x = 5 \cdot (-\frac{2}{5})$
$2x = 1$	$5x = -2$
$-1 + 2x = -1 + 1$	$2 + 5x = -2 + 2$
$2x - 1 = 0$	$5x + 2 = 0$

$(2x-1)(5x+2) = 0$

$10x^2 + 4x - 5x - 2 = 0$

$10x^2 - x - 2 = 0$ ✓

#28

a. $2x^2 - 4x - 3 = 0$

$a = 2$, $b = -4$, $c = -3$

Discriminant $= b^2 - 4ac$

$D = (-4)^2 - 4(2)(-3)$

$D = 16 + 24$

$D = 40 > 0$

Two unequal real solutions

b. $x^2 = 2x - 6$

$-2x + 6 + x^2 = -(2x-6) + 2x - 6$

$x^2 - 2x + 6 = 0$

$a = 1$, $b = -2$, $c = 6$

Discriminant $= b^2 - 4ac$

$D = (-2)^2 - 4(1)(6)$

$D = 4 - 24$

$D = -20 < 0$

Two complex solutions (conjugates)

Chapter 11 Review – Math 101 –

#28

c. $x^2 = 6x - 9$

$-6x + 9 + x^2 = -(6x - 9) + 6x - 9$

$x^2 - 6x + 9 = 0$

$\begin{cases} a = 1 \\ b = -6 \\ c = 9 \end{cases}$

Discriminant $= b^2 - 4ac$

$D = (-6)^2 - 4(1)(9)$

$D = 36 - 36$

$D = 0$

One real solution (Repeated)

#29

a. $4x^2 - x - 1 = 0$

$\begin{cases} a = 4 \\ b = -1 \\ c = -1 \end{cases}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-1)}}{2(4)}$

$x = \dfrac{1 \pm \sqrt{1 + 16}}{8}$

$x = \dfrac{1 \pm \sqrt{17}}{8}$

Either

$x = \dfrac{1 + \sqrt{17}}{8}$, or $x = \dfrac{1 - \sqrt{17}}{8}$

$x \approx \dfrac{1 + 4.123}{8}$	$x \approx \dfrac{1 - 4.123}{8}$
$x \approx \dfrac{5.123}{8}$	$x \approx \dfrac{-3.123}{8}$
$x \approx 0.640375$	$x \approx -0.390375$
$x \approx 0.6$	$x \approx -0.4$

$\{0.6, -0.4\}$

Chapter 11 Review — Math 101 —

#29

b. $1.25x^2 - 0.75x - 3.25 = 0$

$\begin{cases} a = 1.25 \\ b = -0.75 \\ c = -3.25 \end{cases}$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-0.75) \pm \sqrt{(-0.75)^2 - 4(1.25)(-3.25)}}{2(1.25)}$

$x = \dfrac{0.75 \pm \sqrt{0.5625 + 16.25}}{2.5}$

$x = \dfrac{0.75 \pm \sqrt{16.8125}}{2.5}$

Either

$x = \dfrac{0.75 + \sqrt{16.8125}}{2.5}$, or $x = \dfrac{0.75 - \sqrt{16.8125}}{2.5}$

$x \approx \dfrac{0.75 + 4.1003}{2.5}$ | $x \approx \dfrac{0.75 - 4.1003}{2.5}$

$x \approx \dfrac{4.8503}{2.5}$ | $x \approx \dfrac{-3.3503}{2.5}$

$x \approx 1.94012$ | $x \approx -1.34012$

$x \approx 1.9$ | $x \approx -1.3$

$\{1.9, -1.3\}$

c. $\frac{2}{5}x^2 + \frac{1}{5}x - 7 = 0$

$\frac{15}{1}\left[\frac{2}{5}x^2 + \frac{1}{5}x - 7\right] = \frac{15}{1}\cdot 0$

$5.2x^2 + 3.1x - 105 = 0$

$10x^2 + 3x - 105 = 0$

$\begin{cases} a = 10 \\ b = 3 \\ c = -105 \end{cases}$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(10)(-105)}}{2(10)}$

$x = \dfrac{-3 \pm \sqrt{9 + 4200}}{20}$

$x = \dfrac{-3 \pm \sqrt{4209}}{20}$

Either

$x = \dfrac{-3 + \sqrt{4209}}{20}$, or $x = \dfrac{-3 - \sqrt{4209}}{20}$

$x \approx \dfrac{-3 + 64.8768}{20}$ | $x \approx \dfrac{-3 - 64.8768}{20}$

$x \approx \dfrac{61.8768}{20}$ | $x \approx \dfrac{-67.8768}{20}$

$x \approx 3.09384$ | $x \approx -3.39384$

$x \approx 3.1$ | $x \approx -3.4$

$\{3.1, -3.4\}$

Chapter 11 Review — Math 101 —

#30:

a. $f(x) = x^2 - 4.2x + 8.1$, $a = 1$, $b = -4.2$ & $c = 8.1$

Vertex $= \left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ ← $a > 0$, up

$x = \frac{-b}{2a}$

$x = \frac{-(-4.2)}{2(1)}$

$x = \frac{4.2}{2}$

$x = 2.1$

$y = f\left(\frac{-b}{2a}\right)$

$y = f(2.1)$

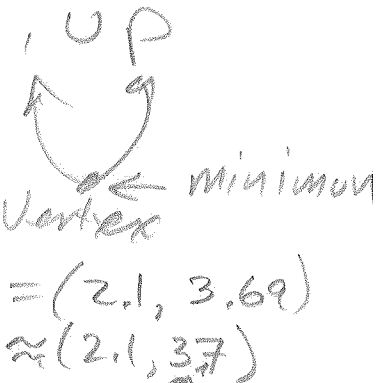
$y = (2.1)^2 - 4.2(2.1) + 8.1$

$y = 4.41 - 8.82 + 8.1$

$y = -4.41 + 8.1$

$y = 3.69$

$y \approx 3.7$ ← minimum value



$= (2.1, 3.69)$

$\approx (2.1, 3.7)$

b. $f(x) = 2.1x^2 + 6.5x - 3.1$

$a = 2.1$, $b = 6.5$, & $c = -3.1$, $a > 0$, up

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(6.5)}{2(2.1)}$

$x = \frac{-6.5}{4.2}$

$x \approx -1.5476$

$x \approx -1.5$

$y = f\left(\frac{-b}{2a}\right)$

$y \approx f(-1.5)$

$y \approx 2.1(-1.5)^2 + 6.5(-1.5) - 3.1$

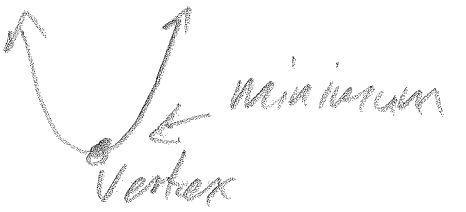
$y \approx 2.1(2.25) - 9.75 - 3.1$

$y \approx 4.725 - 9.75 - 3.1$

$y \approx -5.025 - 3.1$

$y \approx -8.125$

$y \approx -8.1$ ← Minimum value



$\approx (-1.5, -8.1)$

Chapter 11 Review – Math 101 –

#30

c. $f(x) = 3.4x^2 - 4.1x + 1$

$a = 3.4$, $b = -4.1$, & $c = 1$

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = \frac{-b}{2a}$

$x = \frac{-(-4.1)}{2(3.4)}$

$x = \frac{4.1}{6.8}$

$x \approx 0.60294\ldots$

$x \approx 0.6$

$y = f\left(\frac{-b}{2a}\right)$

$y \approx f(0.6)$

$y \approx 3.4(0.6)^2 - 4.1(0.6) + 1$

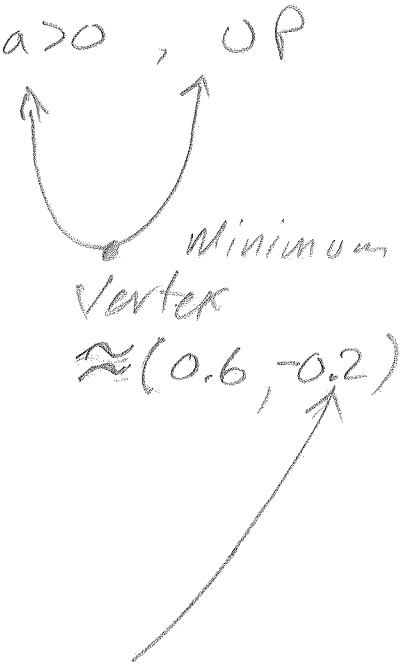
$y \approx 3.4(0.36) - 2.46 + 1$

$y \approx 1.224 - 2.46 + 1$

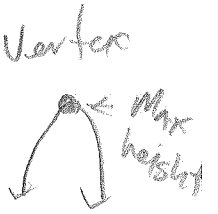
$y \approx -1.236 + 1$

$y \approx -0.236$

$y \approx -0.2$ ← Minimum Value



#31 $s(t) = -16t^2 + 64t + 160$ $a = -16$, $a < 0$,



a. $s(t)$ = height in feet, t seconds after throw

Vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$t = \frac{-b}{2a}$

$t = \frac{-(64)}{2(-16)}$

$t = \frac{64}{32}$

$t = 2$

$y = s\left(\frac{-b}{2a}\right)$

$y = s(2)$

$y = -16(2)^2 + 64(2) + 160$

$y = -16(4) + 128 + 160$

$y = -64 + 128 + 160$

$y = 64 + 160$

$y = 224$ ← Maximum Value

$s(2) = 224$ feet

The maximum height of the ball is 224 feet.

Chapter 11 Review — Math 101 —

#31.

b. height when ball hits the ground $s(t) = 0$ feet

$$0 = -16t^2 + 64t + 160$$

$$-\tfrac{1}{16} \cdot 0 = -\tfrac{1}{16}\left[-16t^2 + 64t + 160\right]$$

$$0 = t^2 - 4t - 10$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 1,\ b = -4,\ c = -10$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$$

$$t = \frac{4 \pm \sqrt{16 + 40}}{2}$$

$$t = \frac{4 \pm \sqrt{56}}{2}$$

either $t = \frac{4 + \sqrt{56}}{2}$, or $t = \frac{4 - \sqrt{56}}{2}$

$t \approx \frac{4 + 7.483}{2}$	$t \approx \frac{4 - 7.483}{2}$
$t \approx \frac{11.483}{2}$	$t \approx \frac{-3.483}{2}$
$t \approx 5.7415$	$t \approx -1.7415$
$\boxed{t \approx 5.7}$	↑ Negative Time??

The ball hits the ground after approximately 5.7 seconds.

c. $s(0) = -16(0)^2 + 64(0) + 160$

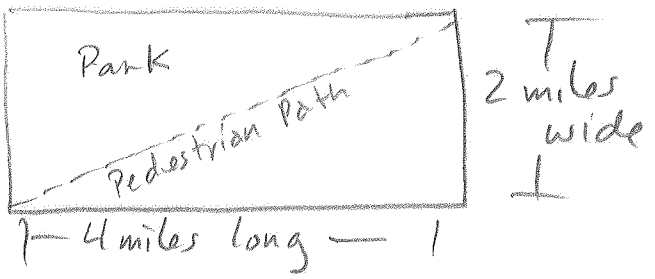
$s(0) = 0 + 0 + 160$

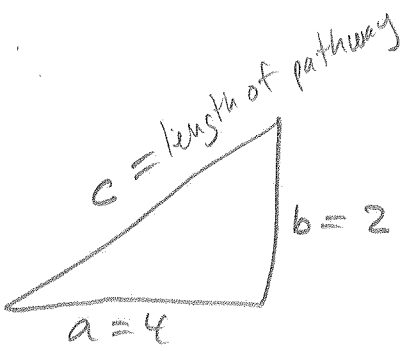
$s(0) = 160$ feet

The ball has not been thrown upward, and the initial height of the ball is 160 feet.

Chapter 11 Review – Math 101 –

#32





$a^2 + b^2 = c^2$

$(4)^2 + (2)^2 = c^2$

$16 + 4 = c^2$

$20 = c^2$

Either

$\sqrt{20} = c$, or $-\sqrt{20} = c$ ← negative length ??

$\sqrt{4}\sqrt{5} = c$

$2\sqrt{5} = c$

$c \approx 2 \cdot (2.36067978...)$

$c \approx 4.472135955...$

$c \approx 4.5$ miles

The length of the pedestrian path is approximately 4.5 miles.

Chapter 11 Review — Math 101 —

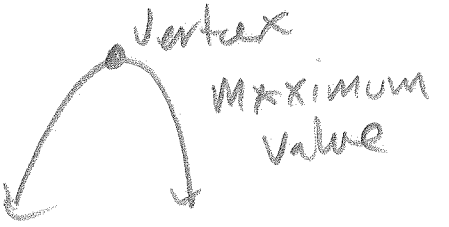
#33 $f(x) = -0.018x^2 + 1.93x - 25.34$

$a = -0.018$

$b = 1.93$

$c = -25.34$

$a < 0$, down



$\text{Vertex} = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\text{Vertex} \approx (53.61, f(53.61))$

x = miles per hour

$f(x)$ = miles per gallon for a Ford Taurus driven at speed x

$$x = \frac{-b}{2a}$$

$$x = \frac{-(1.93)}{2(-0.018)}$$

$x \approx 53.61$ → Rounded to $x \approx 54$

$y = f\left(\frac{-b}{2a}\right)$

$y \approx f(53.61)$

$y \approx -0.018(53.61)^2 + 1.93(53.61) - 25.34$

$y \approx -0.018(2,874.0321) + 103.4673 - 25.34$

$y \approx -51.7326 + 103.4673 - 25.34$

$y \approx 51.7347 - 25.34$

$y \approx 26.3947$

$y \approx 26.4$ → Rounded to $y \approx 26$

The car should be driven at approximately 54 miles per hour to get the optimal miles per gallon of 26 miles per gallon.

Chapter 11 Review – Math 101–

#34.

$A = P(1+r)^t$

Solve for r:

$\$2,916 = \$2500 \cdot (1+r)^2$

$t = 2$ years ← time
$P = \$2,500$ ← principal
$A = \$2,916$ ← Accrued investment
r = annual interest rate

$\frac{2,916}{2,500} = \frac{2,500}{2,500} \cdot (1+r)^2$

$1.1664 = (1+r)^2$

↙ Either \

$\pm\sqrt{1.1664} = 1+r$, or $-\sqrt{1.1664} = 1+r$

$1.08 = 1+r$ | $-1.08 = 1+r$

$-1+1.08 = -1+1+r$ | $-1+(-1.08) = -1+1+r$

$0.08 = r$ | $-2.08 = r$

$8\% = r$ | Negative Interest Rate???

Over two years, an investment of $2,500 will grow to $2,916 at an annual interest rate of 8%.

Chapter 11 Review — Math 101 —

#35

$s(t) = -16t^2 + 140t + 3$, t = seconds after ball was hit, $s(t)$ = ball's height above the ground in feet

$s(t) = 0$ when the ball hits the ground

Solve for t:

$$0 = -16t^2 + 140t + 3$$

$\begin{cases} a = -16 \\ b = 140 \\ c = 3 \end{cases}$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(140) \pm \sqrt{(140)^2 - 4(-16)(3)}}{2(-16)}$$

$$t = \frac{-140 \pm \sqrt{19600 + 192}}{-32}$$

$$t = \frac{-140 \pm \sqrt{19,792}}{-32}$$

$$t = \frac{-140 \pm \sqrt{16}\sqrt{1,237}}{-32}$$

$$t = \frac{-140 \pm 4\sqrt{1,237}}{-32}$$

Either

$$t = \frac{-140 + 4\sqrt{1,237}}{-32} \quad \text{or} \quad t = \frac{-140 - 4\sqrt{1,237}}{-32}$$

$$t \approx \frac{-140 + 4(35.171)}{-32} \qquad t \approx \frac{-140 - 4(35.171)}{-32}$$

$$t \approx \frac{-140 + 140.684}{-32} \qquad t \approx \frac{-140 - 140.684}{-32}$$

$$t \approx \frac{0.684}{-32} \qquad t \approx \frac{-280.684}{-32}$$

$t \approx -0.021$ ← Negative time??

$t \approx 8.771375$

$t \approx 8.8$ seconds

After approximately 8.8 seconds, the ball will hit the ground.

SPWK

19,792
2 | 9,896
2 | 4948
2 | 2474
2 | 1,237

Chapter 11 Review – Math 101

#36:

$$H(x) = \frac{15}{8}x^2 - 30x + 200$$

$H(x)$ = heart rate in beats per minute

x = minutes after a strenuous workout

a. Immediately following the workout $x = 0$.

Find $H(0)$:

$H(0) = \frac{15}{8}(0)^2 - 30(0) + 200$

$H(0) = 0 + 200$

$H(0) = 200$

The heart rate should be 200 beats per minute immediately after the workout.

b. Find x when $H(x) \geq 110$: $x \geq 0$ ← Time after workout is not negative.

$\frac{15}{8}x^2 - 30x + 200 \geq 110$

$-110 + \frac{15}{8}x^2 - 30x + 200 \geq -110 + 110$

$\frac{15}{8}x^2 - 30x + 90 \geq 0$

Solve! $\frac{15}{8}x^2 - 30x + 90 = 0$

$\frac{8}{15} \cdot \left[\frac{15}{8}x^2 - 30x + 90\right] = \frac{8}{15} \cdot 0$

$x^2 - 8 \cdot 2x + 8 \cdot 6 = 0$

$x^2 - 16x + 48 = 0$

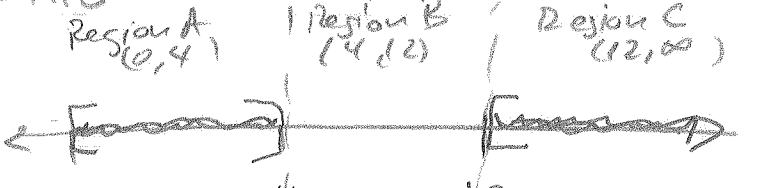
$(x - 12)(x - 4) = 0$

Either

$x - 12 = 0$, or $x - 4 = 0$

$12 + x - 12 = 12 + 0$ | $4 + x - 4 = 4 + 0$

$x = 12$ | $x = 4$



Test $x = 1$

$\frac{15}{8}(1)^2 - 30(1) + 200 \geq 110$

$\frac{15}{8} - 30 + 200 \geq 110$

$231.875 \geq 110$

TRUE!

Test $x = 6$

$\frac{15}{8}(6)^2 - 30(6) + 200 \geq 110$

$\frac{15}{8}(36) - 180 + 200 \geq 110$

$67.5 + 20 \geq 110$

$87.5 \geq 110$

False!

Test $x = 13$

$\frac{15}{8}(13)^2 - 30(13) + 20$

$\frac{15}{8}(169) - 390 + 200$

$316.875 - 190 \geq 110$

$126.875 \geq 110$

TRUE!

The interval of $[0, 4]$ seems reasonable, but after 12 minutes the model seems to breakdown. Solution: $[0, 4] \cup [12, \infty)$.